Bigger Rip with No Dark Energy

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Abstract

By studying a modified Friedmann equation which arises in an extension of general relativity which accommodates a time-dependent fundamental length $L(t)$, we consider cosmological models where the scale factor diverges with an essential singularity at a finite future time. Such models have no dark energy in the conventional sense of energy possessing a truly simple pressure-energy relationship. Data on supernovae restrict the time from the present until the Rip to be generically longer than the current age of the Universe.

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1 Introduction

The cosmic concordance of data from three disparate sources: Cosmic Microwave Background (CMB), Large Scale Structure (LSS) and Type Ia Supernovae (SNeIa) suggests that the present values of the dark energy and matter components, in terms of the critical density, are approximately $\Omega_X \simeq 0.7$ and $\Omega_m \simeq 0.3$.

The equation of state of the dark energy $w = p/\rho$ suggests the possibility that $w < -1$, first studied by Caldwell[1] and subsequently in a number of papers[2, 3].

The conclusion about the make-up of our Universe depends on assuming that general relativity(GR) is applicable at the largest cosmological scales. Although there is good evidence for GR at Solar-System scales[4] there is no independent evidence for GR at scales comparable to the radius of the visible Universe. This involves an extrapolation in scale comparable to that from the weak to the grand unified scale in particle physics (some 13 to 25 orders of magnitude) and could be called the cosmological desert hypothesis. The expansion rate of the Universe, including the present accelerating rate of cosmic expansion can be parameterized in the right-hand side of the Friedmann equation by including a dark energy density term with some assumed time dependence on the scale parameter $a(t)$: $\rho_{DE} \sim a^\beta$ with $\beta = -3(1 + w)$. This is a rather restricted function if we assume that the equation of state w is time-independent. But as soon as we admit that it may depend on time $w(t)$ then the function on the right hand side of the Friedmann equation becomes completely arbitrary, just as does its designation as a dark energy density. We restrict the designation "dark energy" for the normal case where general relativity is assumed at all length scales and the equation of state $w = p/\rho$ is a constant or at most a very simple function of time.

Present data are fully consistent with constant $w = -1$ corresponding to a cosmological constant which we may accommodate on the left hand side of the Friedmann equation describing the expansion rate of the Universe. But cases with $w \neq -1$, including $w < -1$, are still permitted by observations. In this case, there is a choice between cooking up a "dark energy" density with a particular time dependence $\rho_{DE} \sim a^{\beta(t)}$ on the right-hand side of the Friedmann equation or changing the left-hand-side by changing the relationship between the geometry and the matter density, *i.e.* by changing GR.

The case constant $w < -1$ has the interesting outcome for the future of the Universe that it will end in a finite time at a "Big Rip" before which all structure disintegrates[5].

In the present article, we shall study an amalgam of the modification of GR due to Dvali, Gabadadze and Porrati[6] (DGP) and the idea of a Big Rip, in fact here a Bigger Rip. The aim is mere;y to study the range of possibilities in modified Friedmann equations but the results are sufficiently interesting to examine and such modifications may be constrained by observational data.

2 Set up

The DGP gravity[6] arises from considering the four-dimensional gravity which arises from five-dimensional general relativity confined to a brane with three space dimensions. The underlying action is:

$$S = M(t)^3 \int d^5 X \sqrt{G} \mathcal{R}^{(5)} + M_{Planck}^2 \int d^4 x \sqrt{g} R \tag{1}$$

where $\mathcal{R}^{(5)}$ and R are the scalar curvature in 5- and 4- dimensional spacetime respectively, and G and g are the determinant of the 5- and 4- dimensional spacetime metric.

This leads to an interesting modification of GR which embodies a time-dependent length scale $L(t) = M_{Planck}^2/M(t)^3$. For cosmology it is natural to identify, within a coefficient of order one, $L(t)$ with the Hubble length $L(t) = H(t)^{-1}$ at any cosmological time t. Actually we are slightly generalizing the original DGP approach to include time dependence of the length scale $L(t)$.

Taking the four dimensional coordinates to be labeled by $i, k = 0, 1, 2, 3$ leads to the following modification of Einstein's equation, as a result of varying the action Eq.(1):

$$\left(R^{ik} - \frac{1}{2} R g^{ik} \right) + \frac{2\sqrt{G}}{L(t)\sqrt{g}} \left[\left(\mathcal{R}^{(5)ik} - \frac{1}{2} G^{ik} \mathcal{R}^{(5)} \right) \right] = 0 \tag{2}$$

where the specialized notation $[()]$ in Eq.(2) means the following:

$$\int dx [(f(x))] \equiv f'(0)\delta(x). \tag{3}$$

and in Eq.(3) we here identify x with the additional space dimension.

It is interesting to generalize the Schwarzschild solution to this modification of GR[7]. One finds that the modification of the Newton potential at short distances is given by:

$$V(r) = -\frac{Gm}{r} - \frac{4\sqrt{Gm}\sqrt{r}}{L(t)}$$
$$= -\frac{r_g}{2r} - \frac{2\sqrt{2}\sqrt{r_g r}}{L(t)} \tag{4}$$

where $r_g = 2Gm$ is the Schwarzschild radius.

The fractional change in the Newtonian gravitational potential at cosmological time t at orbital distance r from an object with Schwarzschild radius r_g is therefore

$$\left| \frac{\Delta V}{V} \right| = \sqrt{\frac{8r^3}{L(t)^2 r_g}} \tag{5}$$

In the Bigger Rip scenario we will describe the characteristic length $L(t)$ will decrease with time according to

$$L(t) = L(t_0)T(t)^p \tag{6}$$

where the power satisfies $p \geq 1$ ($p < 1$ implies that $L(t)$ would *increase*) and where

$$T(t) = \frac{(t_{rip} - t)}{(t_{rip} - t_0)} \tag{7}$$

in which t_{rip} is the time of the Rip. A bound system will become unbound at a time t_U when the correction to the Newtonian potential becomes large. We make adopt the value of t_U defined from Eq.(5) by

$$\sqrt{\frac{8r^3}{L(t_U)^2 r_g}} = 1 \tag{8}$$

We can rewrite Eq.(8) as:

$$(t_{rip} - t_U) = \frac{1}{\gamma} \left(\frac{8l_0^3}{L_0^2 r_g} \right)^{\frac{1}{2p}} \tag{9}$$

where $\gamma = (t_{rip} - t_0)^{-1}$.

We shall define another later time t_{caus} as the time after which the two objects of a bound system become causally disconnected from t_{caus} until t_{rip}. This is defined by the equation:

$$(t_{rip} - t_{caus}) = \frac{l_0}{c} \left(\frac{a(t_{caus})}{a(t_U)} \right) \tag{10}$$

As an example taking $p = 1$ with the values $L_0 = H_0^{-1} = (14Gy)^{-1} = 1.3 \times 10^{28} cm$ and $\gamma = (20Gy)^{-1}$ we arrive at the entries in the following Table:

Bound system	$l_0(cm)$	r_g(cm)	$(t_{rip} - t_U)$	$(t_{rip} - t_{caus})$
Typical galaxy	5×10^{22}	3×10^{16}	100My	4My
Sun-Earth	1.5×10^{13}	2.95×10^5	2mos	31hr
Earth-Moon	3.5×10^{10}	0.866	2weeks	1hr

Note that the values we find for $(t_{rip} - t_U)$ are consistent with those found in [5]. The corresponding dark energy would have equation of state $w = -1 - \frac{2}{3}\gamma L_0 = -1.466$ which, like that of [5], is now outside of the range allowed by observations[8] if we assume a constant equation of state although it is allowed in the present model with its time-dependence. As another example, with more normal present w, we can increase the time to the Rip to $\gamma = (50Gy)^{-1}$ in which case $w(t_0) = -1.19$ and the Table is modified to:

Bound system	$l_0(cm)$	r_g(cm)	$(t_{rip} - t_U)$	$(t_{rip} - t_{caus})$
Typical galaxy	5×10^{22}	3×10^{16}	250My	7My
Sun-Earth	1.5×10^{13}	2.95×10^5	5mos	2days
Earth-Moon	3.5×10^{10}	0.866	1mo	2hrs

so with the more lengthy wait until the Big Rip the disintegration of structure and causal disconnection occur correspondingly earlier before the eventual Rip.

3   The Bigger Rip

The modified Friedmann equation for DGP gravity is

$$H^2 - \frac{H}{L(t)} = 0 \tag{11}$$

so that we arrive at:

$$\frac{\dot{a}}{a} = H(t) = H(t_0)\frac{1}{T^p} \tag{12}$$

In Eqs.(11,12) we can neglect, for the future evolution, the term $(\rho_M + \rho_\gamma)/(3M_{Planck}^2)$ on the right-hand-side of the modified Friedmann equation. Defining $\gamma = -dT/dt = (t_{rip} - t_0)^{-1}$ gives:

$$\ln a(t) = - \int_1^{T(t)} \frac{dT}{\gamma L(t_0)T^p} \tag{13}$$

and hence, for $p = 1$, which is similar to dark energy with a constant $w < -1$ equation of state:

$$a(t) = T^{-\frac{1}{\gamma L(t_0)}} \tag{14}$$

while for the Bigger Rip case $p > 1$ one finds

$$a(t) = a(t_0) \exp\left[\left(\frac{1}{T^{p-1}} - 1\right)\frac{1}{(p-1)\gamma L(t_0)}\right] \tag{15}$$

Here we see that the scale factor diverges more singularly in T for $p > 1$, hence the designation of *Bigger Rip*. In particular we study the values $p = 2, 3, \cdots$ as alternative to the "dark energy" case $p = 1$.

Inverting Eq.(15) gives:

$$T = [1 + (p-1)\gamma L(t_0) \ln a(t)]^{-\frac{1}{(p-1)}} \tag{16}$$

In this case there is strictly no dark energy, certainly not with a constant equation of state, but we can mimic it with a fictitious energy density ρ_L by noticing that $H^2 \sim T^{-2p}$ and writing

$$\rho_L \sim [1 + (p-1)\gamma L(t_0) \ln a(t)]^{\frac{2p}{(p-1)}} \tag{17}$$

If we use Eqs.(15) and (17) in conservation of energy

$$\frac{d}{dt}(\rho_L a^3) = -p\frac{d}{dt}(a^3) = -w_L(t)\rho_L \frac{d}{dt}(a^3) \tag{18}$$

we find a time-dependent $w_L(t)$ for the "fictitious" dark energy

$$w_L(t) = -1 + \frac{2}{3}\frac{dL(t)}{dt} \tag{19}$$

$$= -1 - \frac{2}{3}\frac{p\gamma L(t_0)}{1 + (p-1)\gamma L(t_0)\ln a(t)} \tag{20}$$

so the effective $w_L(t)$ has the limiting values $w_L(t_0) = -1 - \frac{2}{3}p(\gamma L(t_0))$ and $w_L(t_{rip}) = -1$.

We may check consistency with the space-space components of Einstein's equations which are

$$\frac{\ddot{a}}{a} = -\frac{1}{2}\frac{\dot{a}^2}{a^2} - 4\pi G p \tag{21}$$

which leads to

$$\dot{H} = -4\pi G(\rho_L + p_L) = -4\pi\rho_L(1 + w_L) \tag{22}$$

so that with $H = L^{-1}$ and $\rho_L = 3H/(8\pi GL)$ we find a $w_L(t)$ consistent with Eq.(20).

Keeping the value $L_0 = H_0^{-1} = (14Gy)^{-1} = 1.3 \times 10^{28}$ cm and putting $\gamma = (20Gy)^{-1}$ and $p = 2$ we arrive at the entries in the following Table:

Bound system	$l_0(cm)$	r_g(cm)	$(t_{rip} - t_U)$	$(t_{caus} - t_U)$
Typical galaxy	5×10^{22}	3×10^{16}	$2.37Gy$	1.14Gy
Sun-Earth	1.5×10^{13}	2.95×10^5	$9.6 \times 10^4 y$	$7y.$
Earth-Moon	3.5×10^{10}	0.866	$2.5 \times 10^4 y$	$6mos.$

Note that for the $p = 2$ case we have tabulated the difference $(t_{caus} - t_U)$ rather than $(t_{rip} - t_{caus})$ because in this case the expansion is so rapid.

Next we turn to observational constraints on the parameters L_0 and γ for $p = 2$.

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4   Observational Constraints

In the previous section, we discussed the future universe in the model. In this section, we discuss constraints on model parameters from SNeIA data. To discuss the constraint, we have to include other component such as cold dark matter (CDM) and baryon. Including all components, we can write the Friedmann equation as [9]

$$H^2 + \frac{k}{a^2} = \left(\sqrt{\frac{\rho_m}{3M_{\mathrm{Planck}}^2} + \frac{1}{4L^2}} + \frac{1}{2L} \right)^2 \tag{23}$$

If we define the density parameter $\Omega_m \equiv \rho_m/\rho_{\mathrm{crit}} = \rho_{m0}(1+z)^3$, we can rewrite Eq. (23) as

$$H^2 = H_0^2 \left[\Omega_k(1+z)^2 + \left(\sqrt{\Omega_L} + \sqrt{\Omega_L + \Omega_m(1+z)^3} \right)^2 \right] \tag{24}$$

where Ω_k and Ω_L are defined as

$$\Omega_k \equiv \frac{-k}{H_0^2}, \qquad \Omega_L \equiv \frac{1}{4L^2 H_0^2}. \tag{25}$$

Thus at the present time, we have the relation among the density parameters,

$$\Omega_k + \left(\sqrt{\Omega_L} + \sqrt{\Omega_L + \Omega_m} \right)^2 = 1. \tag{26}$$

To obtain a constraint from the SNeIa observations, we assume that red-shift dependence of Ω_L is as in Eq. (17) and that L is dependent on time as Eq. (6); thus we can say that we consider a new component ρ_L which is defined as Eq. (17).

Now we discuss the constraint on this model from SNeIa data using recent result [8]. In Fig. 1, we show contours of 95 and 99 % C.L. in Ω_L-Ω_m plane for $\gamma L_0 = 0$ (which is the constant L case), 0.5 and 1 with $p = 2$. In the figure, we also plot the line for the flat universe. Notice that the line is different from the standard case because we have the modified Friedmann equation in this model. To obtain the constraint, we marginalize the Hubble parameter dependence by minimizing χ^2 for the fit.

In Fig. 2, we show the constraint on Ω_{m}-γL_0 plane assuming the flat universe ($\Omega_k = 0$). If we take the value $\Omega_{\mathrm{m}} = 0.3$, we can find an upper limit for γL_0 which is $\gamma L_0 \lesssim 0.7$. This implies that the time remaining from now until the Rip is constrained to be generically at least somewhat longer than the current age. If we make L_0 larger than the length corresponding to the age of the Universe then the upper bound on γ diminishes and hence the time until the Rip increases.

We note that because the effective equation of state $w(t)$ is varying with time its present value $w(t_0)$ can be more negative than allowed by constraints derived from assuming constant w. Our constraint on $\gamma L_0 < 0.7$ permits $w(t_0) = -1 - \frac{2}{3}p\gamma L_0$ to be as negative as $w(t_0) = -1.9$ for $p = 2$. Assuming constant w, on the other hand, gives [8] $w > -1.2$.

5 Discussion

The present article is a natural sequel to our previous paper [3] about dark energy in which it was pointed out that no amount of observational data can, by itself, tell us the fate of dark energy if we allow for an arbitrarily varying equation of state. The three possibilities listed were: there may firstly be a Big Rip, or secondly dark energy may dominate but with an infinite lifetime or thirdly the dark energy may eventually disappear leaving a matter-dominated Universe. Given that observational data are insufficient, only a successful and convincing theory may inform us confidently of the future of the Universe.

The Big Rip was the most exotic of the fates and there seemed tied to a phantom $w < -1$ dark energy. However, here we have studied a Bigger Rip, in which the scale factor is even more divergent at a future finite time than for the Big Rip, which is achieved by modifying gravity and omitting dark energy. In the model, as with the phantom case, structures become unbound and subsequently their components become causally disconnected before the Universe is torn apart in the Rip.

Work by other groups [10] has recently suggested that quite different values of cosmic parameters can be acceptable if one relaxes the most conventional and conservative assumptions of general relativity at all length scales and a dark energy with a constant equation of state.

Acknowledgments

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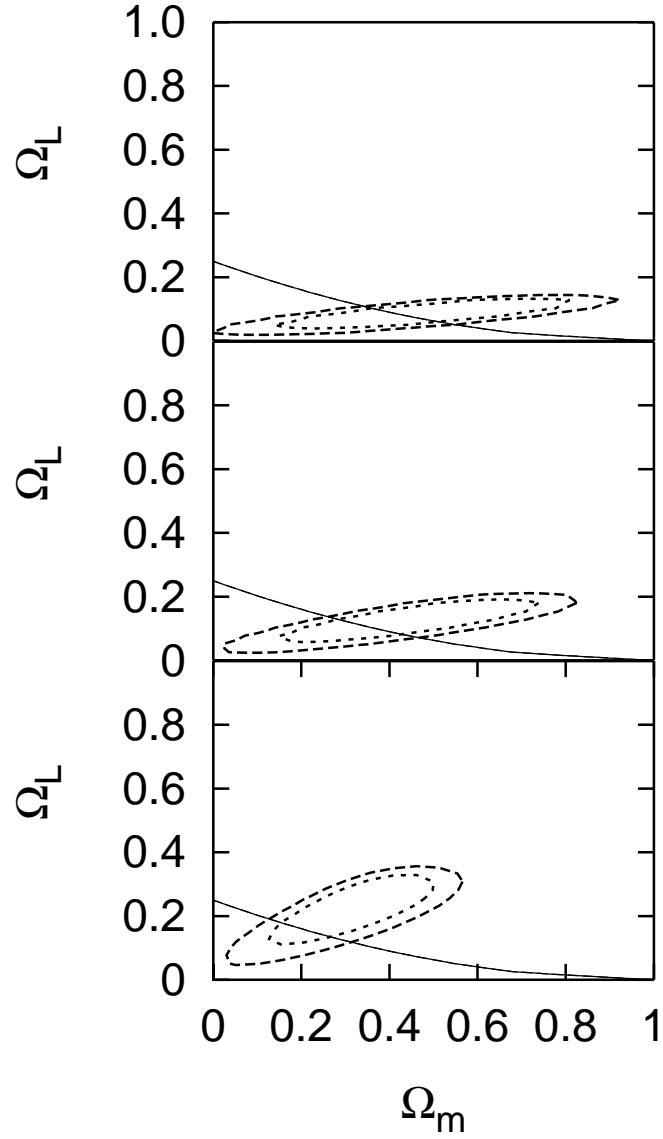J.M. Virey *et al.* `astro-ph/0407452`.

Figure 1: Constraint from SNeIa observation in Ω_L-Ω_m plane for $\gamma L_0 = 0$ (bottom), $\gamma L_0 = 0.5$ (middle) and $\gamma L_0 = 2$ (top). Contours are for 95 % (dotted line) and 99 % (dashed line) C.L. constraints respectively. The solid line indicates parameters which give a flat universe.
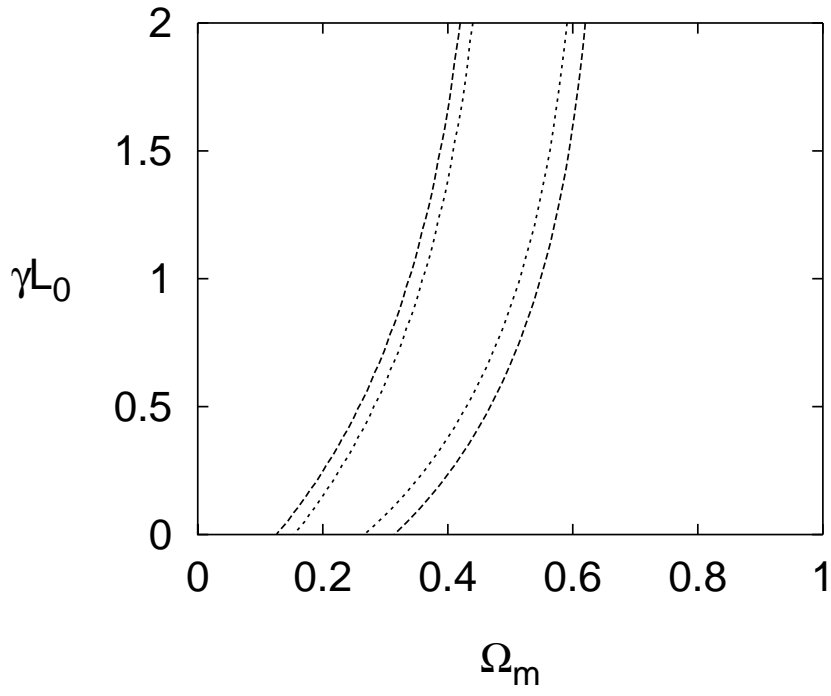
Figure 2: Constraint from SNeIa observation in Ω_{r_c}-$\Omega_{\rm m}$ plane. Contours are for 95 % (dotted line) and 99 % (dashed line) C.L. constraints respectively. In this figure, we assume a flat universe.